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# Investigating Subclasses of Abstract Dialectical Frameworks

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February 20, 2018

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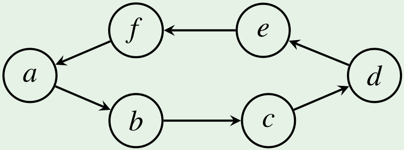
# Introduction

- Argumentation has recently become a main topic within artificial intelligence
  - crucial importance in other fields of science
  - connection to other areas of AI
- Argumentation: negotiating beliefs among agents
  - Argument presents beliefs and reasonable justifications

# Introduction

- Abstract Argumentation frameworks (AFs ) [Dung, 1995]
  - A set of arguments
  - A binary relation between arguments which represents conflicts (attacks)

## Example




- Semantics: Methods used to clarify the acceptance of arguments
- Extension: set of jointly accepted arguments

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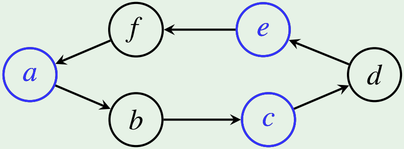


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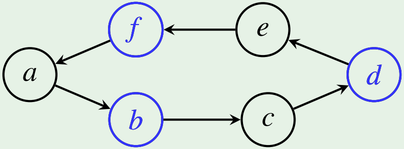


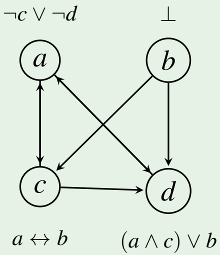

$stb(F) = \{\{a, c, e\}, \{b, d, f\}\}$

- Semantics: Methods used to clarify the acceptance of arguments
- Extension: set of jointly accepted arguments

# Introduction

- Abstract Dialectical Frameworks (ADFs) [Brewka and Woltran, 2010, Brewka et al., 2013]
  - Unify several generalizations of AFs
  - Express relations between arguments beyond simple attack [group attack, support, group support,...]
  - This is modeled via acceptance conditions

## Example



# Motivation

## Vibrant research on AFs

- Interesting subclasses of AFs are identified
- Properties of these subclasses are studied
- Coincidence of semantics for a subclass of AFs is clarified
- Expressivity of different semantics of AFs is investigated

## Question

How do these carry over to ADFs?

# Motivation

## Main Contributions

- Defining subclasses of ADFs
  - Coincidence of semantics
  - Properties
- Studying the expressiveness of different argumentation formalisms from the perspective of realizability
- Analyzing the effect of cycles on the performance of solvers for ADFs

# Outline

# Background

### Definition

An *argumentation framework* (AF) is a pair $(A, R)$ s.t.

- $A$ is a finite set of arguments
- $R \subseteq A \times A$ is a binary relation representing attacks between arguments

- $S \in \mathit{cf}(F)$ if there is no $a, b \in S$ s.t. $(a, b) \in R$
- An argument $a \in A$ is *defended* by $S \subseteq A$ (in $F$) if $\forall c \in A$: if $(c, a) \in R$ then $\exists b \in S$ s.t. $(b, c) \in R$
- $\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$

# Background

## Semantics of AFs

Given an AF $F = (A, R)$. A set $S \in cf(F)$ is

- $S \in adm(F)$ if $S \subseteq \Gamma_F(S)$
- $S \in pref(F)$ if $S$ is $\subseteq$-maximal admissible
- $S \in comp(F)$ if $S = \Gamma_F(S)$
- $S \in grd(F)$ if $S$ is the $\subseteq$-least fixed point of $\Gamma_F(S)$
- $S \in stb(F)$ if $\forall a \in A$: $\exists b \in S$ s.t. $(b, a) \in R$
- $S \in naive(F)$ if $S$ is $\subseteq$-maximal conflict-free

# Background

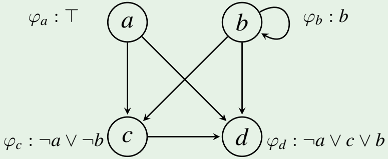
## Definition

An *abstract dialectical framework* (ADF) is a tuple $D = (S, L, C)$ where

- $S$ is a set of nodes (argument, statement, positions)
- $L \subseteq S \times S$ is a set of links
- $C = \{\varphi_s\}_{s \in S}$ is a set of propositional formulas (acceptance conditions)

## Example

$D = (\{a, b, c, d\}, \{(a, c), (a, d), (c, d), (b, b), (b, c), (b, d)\}, \{\varphi_a : \top, \varphi_b : b, \varphi_c : \neg a \vee \neg b, \varphi_d : \neg a \vee c \vee b\})$



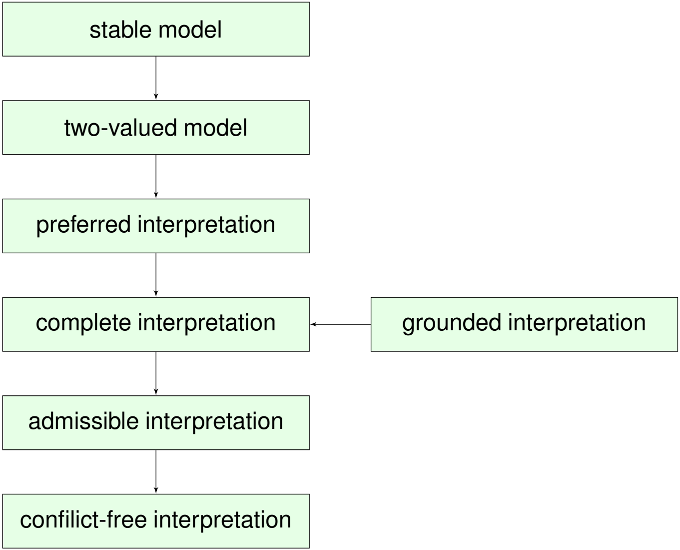
# Background

## Semantics of ADFs

Given an ADF $D$. An interpretation $v$ is

- $v \in \mathit{adm}(D)$ if $v \leq_i \Gamma_D(v)$
- $v \in \mathit{pref}(D)$ if $v$ is $\leq_i$-maximal admissible
- $v \in \mathit{comp}(D)$ if $v = \Gamma_D(v)$
- $v$ is $\mathit{grd}(D)$ if $v$ is the $\leq_i$-least fixed point of $\Gamma_D(v)$
- $v \in \mathit{mod}(D)$ if $v$ is a two-valued interpretation and $v = \Gamma_D(v)$
- $v \in \mathit{stb}(D)$ if $V$ is a model of $D$ and $v^t = w^t$, in which $w$ is the grounded interpretation of $D^t = (v^t, L \cap (v^t \times v^t), \{\varphi_s[p/\bot \ : \ v(p) = f]\}_{s \in v^t})$
- $v \in \mathit{cf}(D)$ if for each $s \in S$; $v(s) = t$ implies $\varphi_s^v$ is satisfiable and $v(s) = f$ implies $\varphi_s^v$ is unsatisfiable

# Background


stable model
two-valued model
preferred interpretation
complete interpretation
grounded interpretation
admissible interpretation
confilict-free interpretation

# Background

### Definition

Given an ADF $D = (S, L, C)$. A link $(r, s) \in L$ is

- attacking in $D$ if $\forall v \in V_2$, $v(\varphi_s) = f$ implies $v|_t^r(\varphi_s) = f$
- supporting in $D$ if $\forall v \in V_2$, $v(\varphi_s) = t$ implies $v|_t^r(\varphi_s) = t$
- dependent if it is neither attacking nor supporting
- redundant if it is both attacking and supporting

### Definition

Given an ADF $D = (S, L, C)$. Let $L^+$ be the set of all support links and $L^-$ be the set of all attack links of $L$. $D$ is named a bipolar ADF (BADF) iff $L = L^+ \cup L^-$.

# Investigating Subclasses of ADFs

**Theorem [Dung, 1995]**

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

**Question**

Does there exist a subclass of ADFs in which different semantics collapse to the same set of interpretations?

# Investigating Subclasses of ADFs

## Theorem [Dung, 1995]

Every well-founded AF has exactly one complete extension which is grounded, preferred and stable.

## Question

Does there exist a subclass of ADFs in which different semantics collapse to the same set of interpretations?

## Definition

A finite ADF $D = (S, L, C)$ is named *acyclic* if its corresponding directed graph does not contain any directed cycle.

## Theorem

An acyclic ADF $D = (S, L, C)$ has exactly one complete interpretation which is grounded, two-valued model, preferred and stable.

# Investigating Subclasses of ADFs

**Definition**

Given an AF $F = (A, L)$.

- It is named *coherent* whenever *pref*$(F)$ = *stb*$(F)$
- It is called *relatively grounded* if *grd*$(F) = \bigcap$ *pref*$(F)$

**Theorem [Coste-Marquis et al., 2005]**

Given a symmetric AF $F$. $F$ is coherent and relatively grounded.

# Investigating Subclasses of ADFs

**Question**

Is every symmetric ADF coherent?

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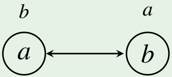
## Example

Given an ADF, $D = (\{a, b\}, \{\varphi_a : b, \varphi_b : a\})$



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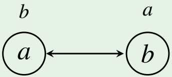


$$\textit{pref}(D) = \{\{a \mapsto t, b \mapsto t\}, \{a \mapsto f, b \mapsto f\}\}$$

$$\textit{mod}(D) = \{\{a \mapsto t, b \mapsto t\}, \{a \mapsto f, b \mapsto f\}\}$$

$$\textit{stb}(D) = \{\{a \mapsto f, b \mapsto f\}\}$$

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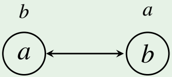



$$pref(D) = \{\{a \mapsto t, b \mapsto t\}, \{a \mapsto f, b \mapsto f\}\}$$

$$mod(D) = \{\{a \mapsto t, b \mapsto t\}, \{a \mapsto f, b \mapsto f\}\}$$

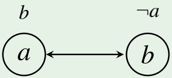
$$stb(D) = \{\{a \mapsto f, b \mapsto f\}\}$$

$$pref(D) = mod(D) \wedge mod(D) \neq stb(D)$$

# Investigating Subclasses of ADFs

## Example

Given an ADF, $D = (\{a, b\}, \{\varphi_a : b, \varphi_b : \neg a\})$




$$pref(D) = \{\{a \mapsto u, b \mapsto u\}\}$$

$$mod(D) = \{\}$$

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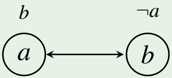


$$pref(D) = \{\{a \mapsto u, b \mapsto u\}\}$$

$$mod(D) = \{\}$$

$$stb(D) = \{\}$$

$$pref(D) \neq mod(D) \wedge mod(D) = stb(D)$$

# Investigating Subclasses of ADFs

**Definition**

Given an ADF $D = (S, L, C)$.

- It is called semi-coherent if $\mathit{pref}(D) = \mathit{mod}(D)$
- It is called weak-coherent if $\mathit{mod}(D) = \mathit{stb}(D)$

# Investigating Subclasses of ADFs

**Definition**

Given an ADF $D = (S, L, C)$.

- It is called semi-coherent if *pref*$(D)$ = *mod*$(D)$
- It is called weak-coherent if *mod*$(D)$ = *stb*$(D)$

**Theorem**

Symmetric ADFs are neither weak-coherent nor semi-coherent.

# Investigating Subclasses of ADFs

### Definition

Given a symmetric ADF $D = (S, L, C)$.

- Attack symmetric ADF (ASADF): all links are attacking ($L = L^+$)
- Acyclic support symmetric ADF (ASSADF): $D$ is a symmetric BADF and does not contain any support cycle

# Investigating Subclasses of ADFs

## Definition

Given a symmetric ADF $D = (S, L, C)$.

- Attack symmetric ADF (ASADF): all links are attacking ($L = L^+$)
- Acyclic support symmetric ADF (ASSADF): $D$ is a symmetric BADF and does not contain any support cycle

## Theorem

ASADFs and ASSADFs are weak-coherent, but not semi-coherent.

# Investigating Subclasses of ADFs

| | Acyclic ADFs | Symmetric ADFs | ASADFs | ASSADFs | CADFs |
|---|---|---|---|---|---|
| *cf* $=$ *adm* | – | – | – | – | – |
| *adm* $=$ *comp* | – | – | – | – | $\surd$ |
| *pref* $=$ *mod* (semi-coherent) | $\surd$ | – | – | – | – |
| *mod* $=$ *stb* (weak-coherent) | $\surd$ | – | $\surd$ | $\surd$ | – |
| *comp* $=$ *pref* | $\surd$ | – | – | – | – |
| *stb* $= \{$*grd*$\}$ | $\surd$ | – | – | – | – |
| relatively grounded | $\surd$ | – | – | – | – |
| coherent | $\surd$ | – | – | – | – |

# Realizability and Expressivity

**Question**

Which set of sets can be the outcome of the evaluation of a formalism under semantics $\sigma$?

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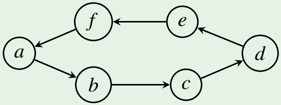

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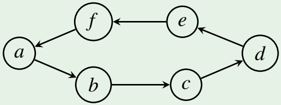


- $\exists F \in AFs$ s.t. $\mathbb{S} = adm(F)$?

# Realizability and Expressivity

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Which set of sets can be the outcome of the evaluation of a formalism under semantics $\sigma$?

### Example

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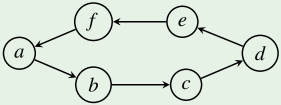
- $\exists F \in AFs$ s.t. $\mathbb{S} =$ *stb*$(F)$? Yes




- $\exists F \in AFs$ s.t. $\mathbb{S} =$ *adm*$(F)$? No

# Realizability and Expressivity

### Definition

The *signature* $\Sigma^{\sigma}_{\mathcal{F}}$ of a formalism $\mathcal{F}$ w.r.t. semantics $\sigma$ is defined as:

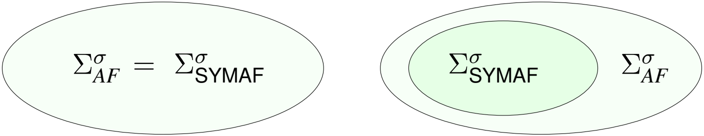
$$\Sigma^{\sigma}_{\mathcal{F}} = \{\sigma(kb) \mid kb \in \mathcal{F}\}$$

### Question

What is the expressive power of semantics of different formalisms via realizability?

# Expressivity of SYMAFs and AFs

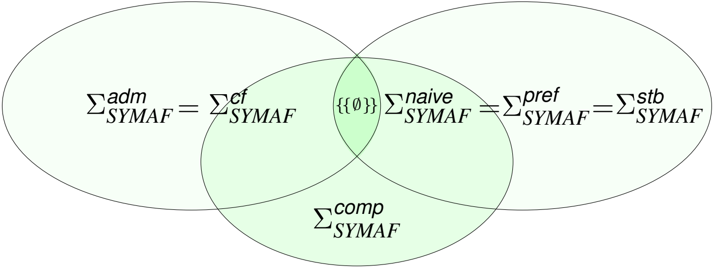
- Relations between signatures of SYMAFs and AFs for $\sigma \in \{adm, cf, naive, pref, stb, comp, grd\}$:




for $\sigma \in \{cf, naive, grd\}$ — for $\sigma \in \{adm, pref, stb, comp\}$

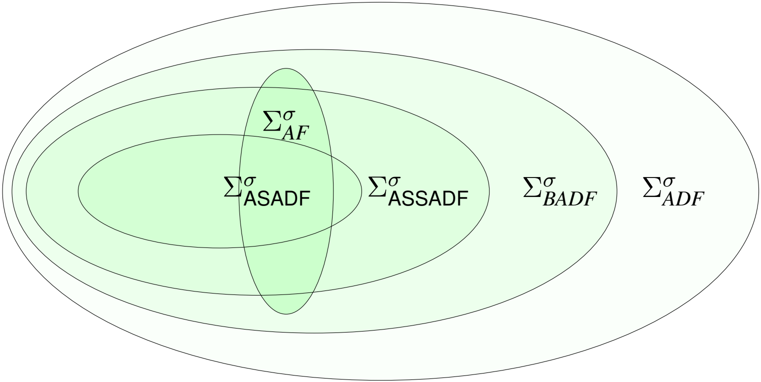
# Expressivity of Symmetric AFs

- Relations between signatures of SYMAFs for $\sigma \in \{adm, cf, naive, pref, comp, stb\}$:



# Expressivity of Subclasses of ADFs

- Expressivity of subclasses of ADFs for $\sigma \in \{adm, pref, comp\}$:




- $\Sigma^{stb}_{AF} \subsetneq \Sigma^{stb}_{\text{ASADF}} \subseteq^{stb}_{\text{ASSADF}} \subseteq \Sigma^{stb}_{BADF} = \Sigma^{stb}_{ADF}$

# Experiments on Subclasses of ADFs

- Generator by which acyclic ADFs, ASADFs and ASSADFs are produced
- The effect of cycles on the performance of solvers for ADFs
  - Credulous acceptance
  - Skeptical acceptance
- Solvers: DIAMOND, goDIAMOND, YADF and QADF

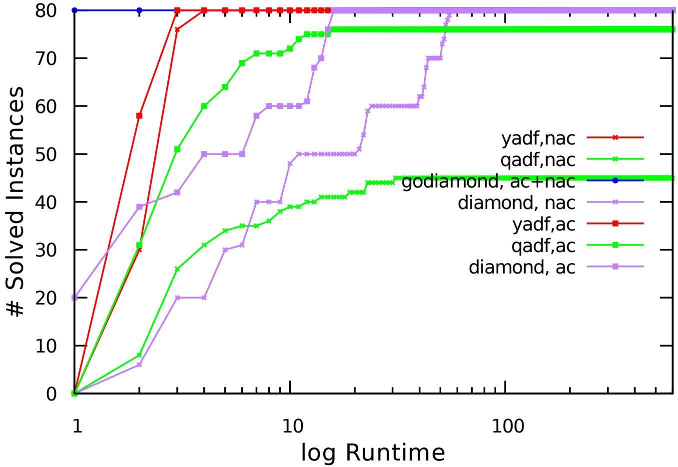
# Experiments on Subclasses of ADFs

- Credulous acceptance under admissible semantics

| | | Time-outs | Mean |
|---|---|---|---|
| *adm*-acyclic | DIAMOND | 0 | 5.2813 |
| | goDIAMOND | 0 | 0.0667 |
| | YADF | 0 | 1.5727 |
| | QADF | 4 | 3.1635 |
| *adm*-non-acyclic | DIAMOND | 0 | 17.641 |
| | goDIAMOND | 0 | 0.202 |
| | YADF | 0 | 2.1679 |
| | QADF | 35 | 5.5060 |

# Experiments on Subclasses of ADFs

- Number of acyclic and non-acyclic ADF-credulous-acceptance problems solved by solvers:



# Experiments on Subclasses of ADFs

- Skeptical acceptance under preferred semantics:

| | | Time-outs | Mean |
|---|---|---|---|
| *pref*-acyclic | DIAMOND | 80 | – |
| | goDIAMOND | 0 | 0.144 |
| | YADF | 57 | 90.696 |
| | QADF | 80 | – |
| *pref*-non-acyclic | DIAMOND | 80 | – |
| | goDIAMOND | 8 | 1.2838 |
| | YADF | 40 | 126.128 |
| | QADF | 80 | – |

# Summary

- Reformulate and prove Dung's Fundamental lemma for ADFs
- Specify a subclass of ADFs in which different semantics can collapse to the same set of interpretations
- Clarify whether symmetric ADFs are coherent and relatively grounded
- Study properties of acyclic ADFs, symmetric ADFs, ASADFs, ASSADFs and CADFs
- Expressiveness of different argumentation formalisms from the perspective of realizability
- Provide a generator to produce subclasses of ADFs
- Use this generator to analyze the effect of cycles on the performance of solvers for ADFs

# Future Work

- Further ADF semantics are introduced by [Polberg, 2016]. How these semantics behave in the subclasses studied in this thesis?
- Studying the computational complexity of the subclasses of ADFs
- Answering open questions in regard to expressiveness

$$\Sigma^{stb}_{\text{ASADF}} \subsetneq^{?} \Sigma^{stb}_{\text{ASSADF}} \subsetneq^{?} \Sigma^{stb}_{BADF}$$

- The results of our experiments suggest that solvers for ADFs can/should be optimized

Thank you for your attention!

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